UNIVERSITY OF WATERLOO

RELATION BETWEEN THE STATIC AND DYNAMIC MODULUS OF ELASTICITY, EXPERIMENTALLY MEASURED

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OUTLINE

- INTRODUCTION
- BACKGROUND
- METHODOLOGY
- RESULTS
- CONCLUSION

ELASTIC MODULUS or YOUNG'S MODULUS

- CONSTANT STRESS/STRAIN

$$\sigma_{nx} = E * \varepsilon_x$$

$$E = \frac{\sigma_{nx}}{\varepsilon_x}$$

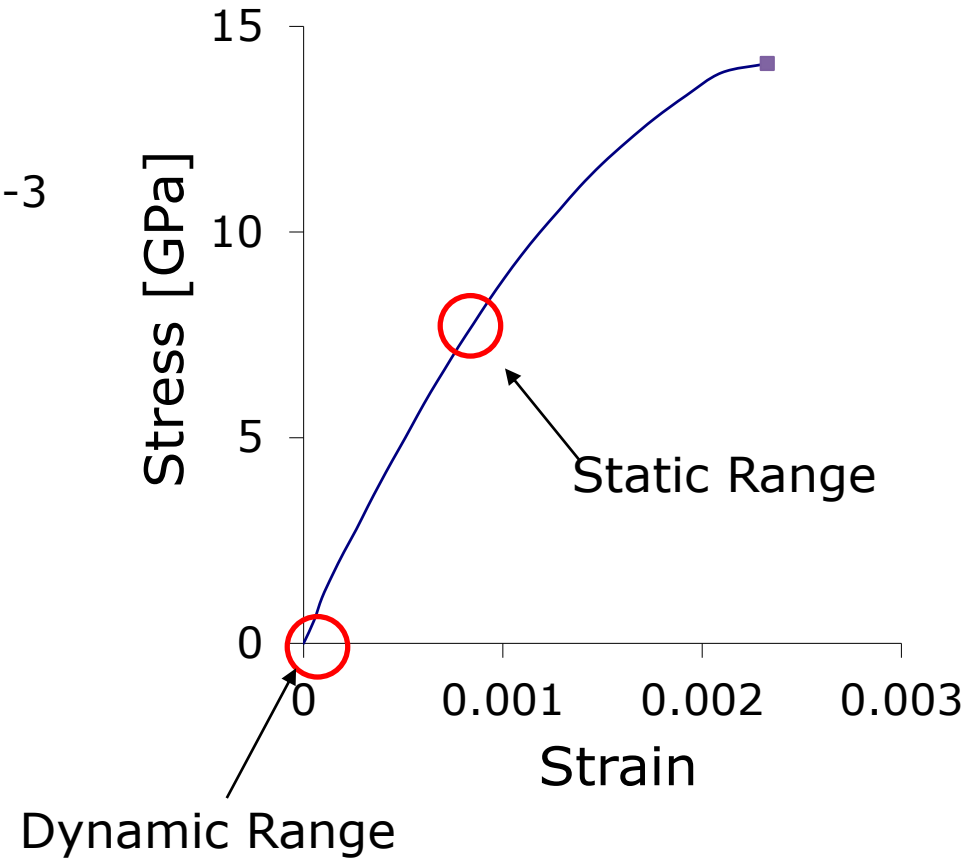
- σ_{nx} = Stress direction strain$_{(x)}$
- ε_x = Strain

STATIC MODULUS (E_S)

- Strain range: 10^{-4} to 10^{-3}
- 5 - 50 [Gpa]

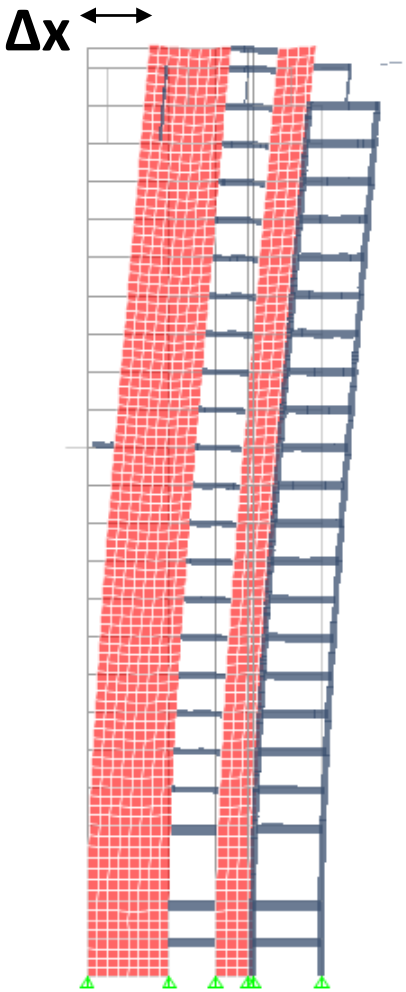
DYNAMIC MODULUS (E_D)

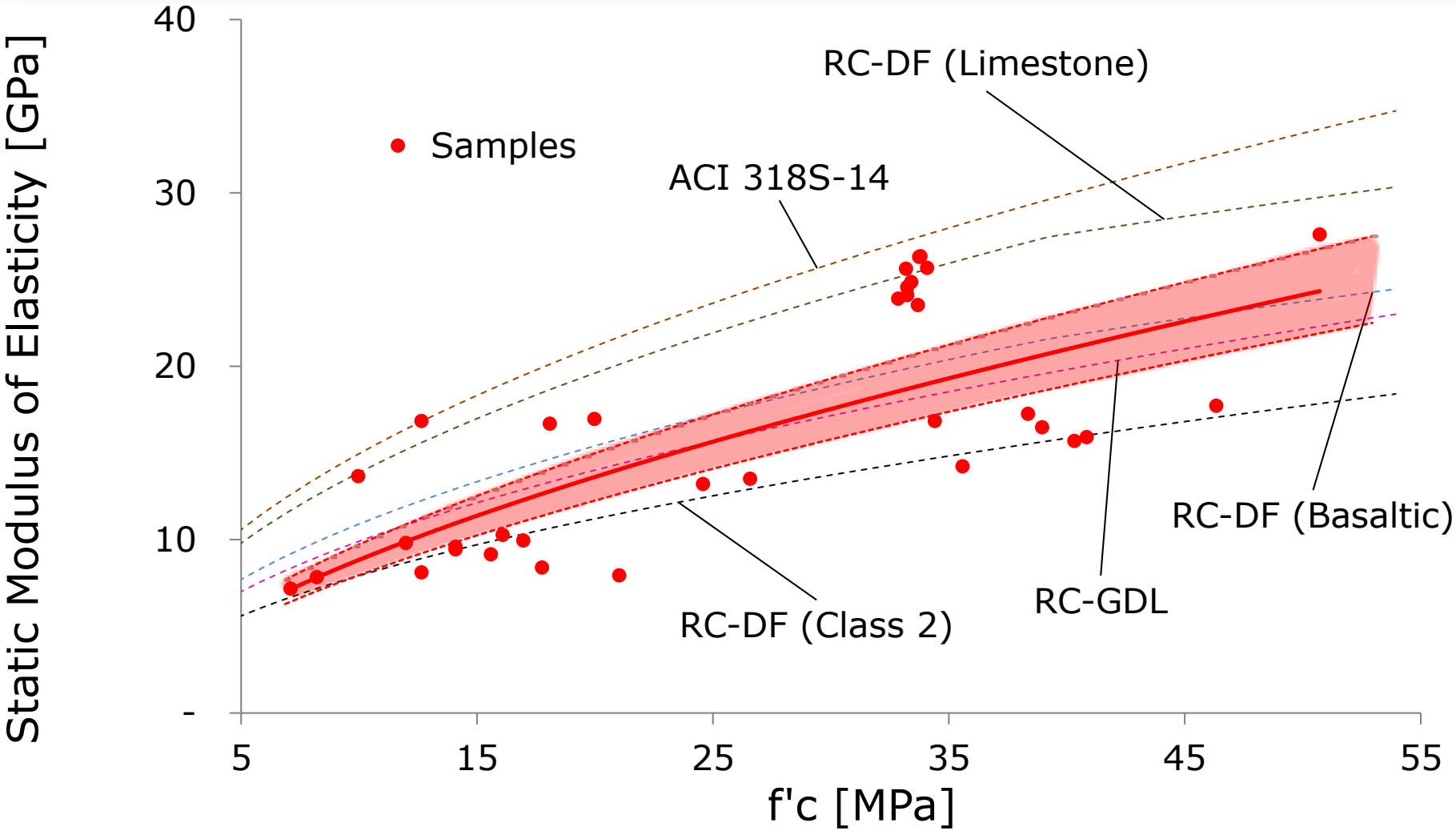
- Strain levels less than: $\sim 10^{-7}$
- $E_D > E_S$



STRUCTURAL RELEVANCE

- Static modulus
- Compute Deformations
- Building Codes [GPa]:
  - $80\sqrt{f'_c}$
  - $150\sqrt{f'_c}$
- On site quality check
  - Cost
  - Time Consuming




Static Modulus of Elasticity [GPa]
40
30
20
10
-
5
15
25
35
45
55
f'c [MPa]
Samples
ACI 318S-14
RC-DF (Limestone)
RC-DF (Basaltic)
RC-GDL
RC-DF (Class 2)

OBJECTIVE

- Function Relationship $E_s(E_d)$
- Cost effective method to evaluate E_s

SCOPE

- 34 concrete cylinders
- Randomly Selected

STATIC MEASURE

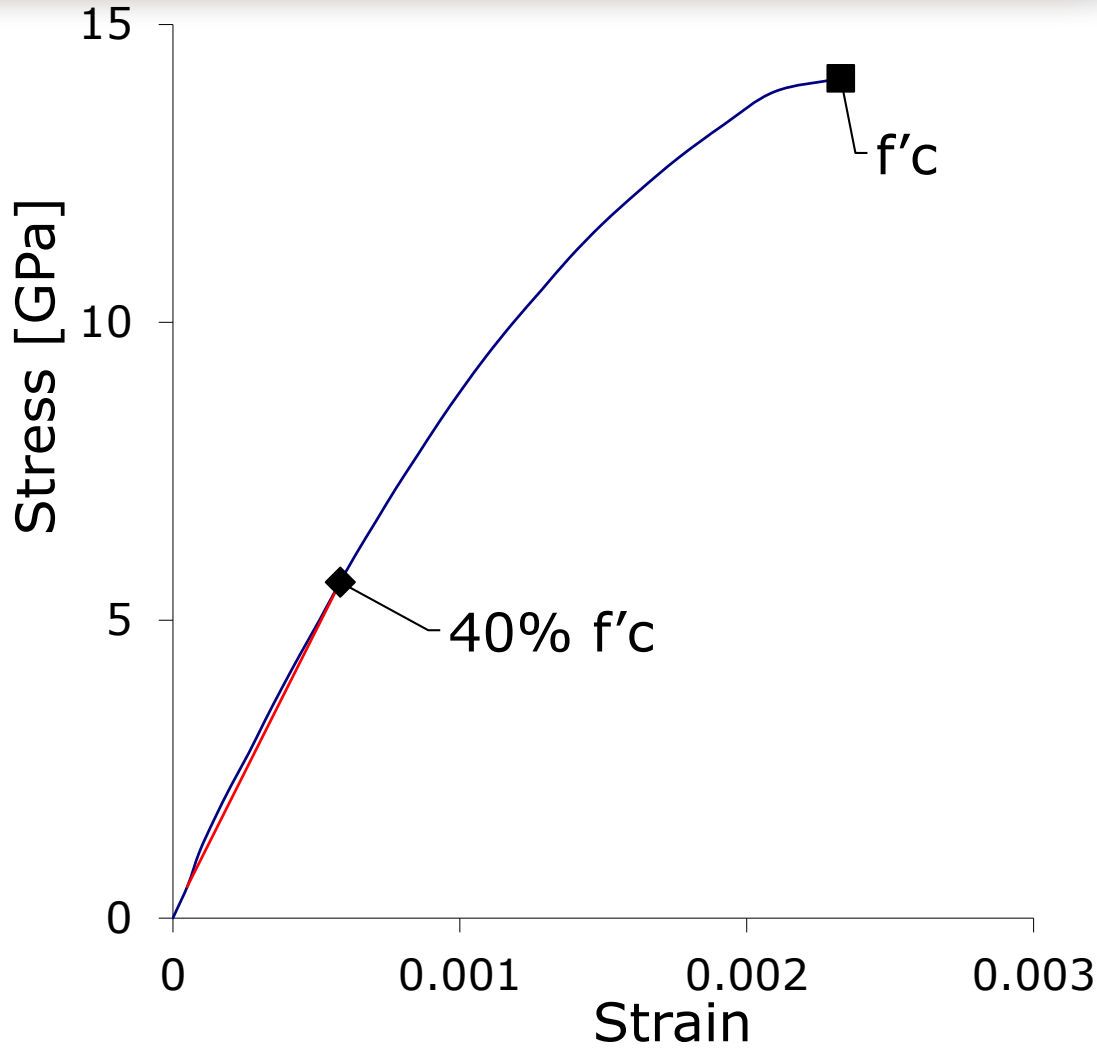
ASTM C 469-14

- $E = \frac{S_2 - S_1}{\epsilon_2 - 50*10^{-6}}$

S_2= Stress 40% f'c

S_1= Stress at ε_1 $(50 * 10^{-6})$

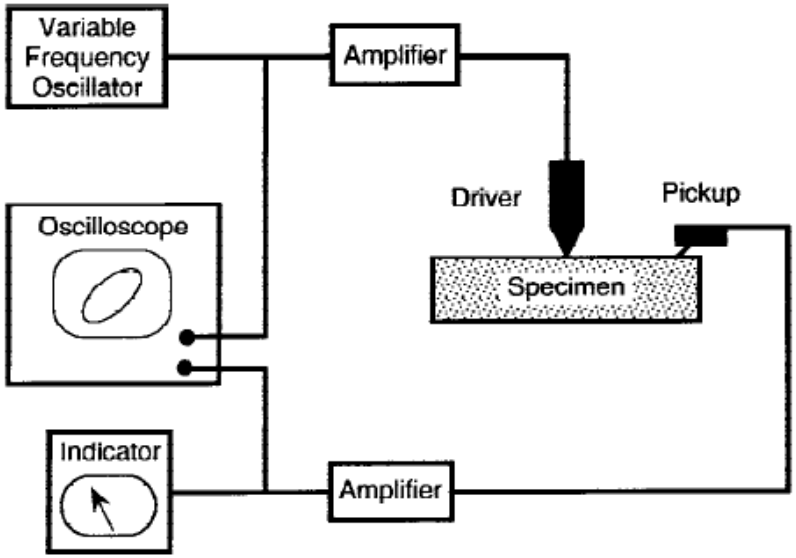
ε_2 = Strain at "S_2"



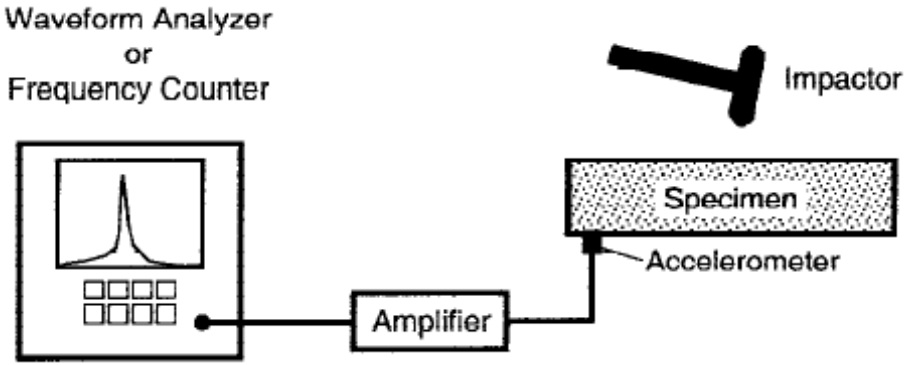
DYNAMIC MEASURE

ASTM C 215-14

- Force Resonance Method

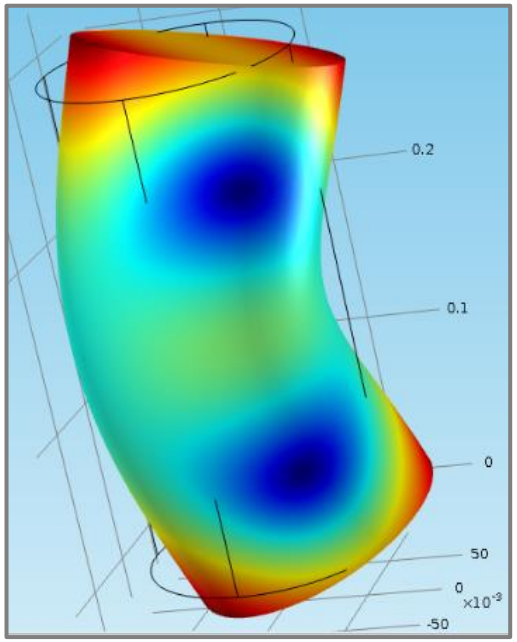


- Impact Resonance Method

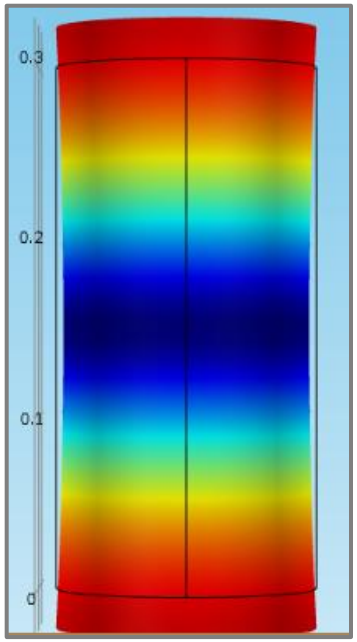


DYNAMIC MEASURE

ASTM C 215-97



Transverse Mode



Longitudinal Mode

WAVE PROPAGATION

P-waves

- $v_p = \sqrt{\frac{M}{\rho}} = \sqrt{\frac{E_D*(1-v)}{\rho*(1+v)*(1-2v)}}$

- Phase Velocity $v_p = f\ \lambda$

  $v_p = f_n\ *(2\ L)$

(Richart, Hall, & Woods, 1970).

PREVIOUS RELATIONS

- $E_S = 0.83\ E_D$ Lydon & Balendran (1986)
- $E_S = 0.23\ {E_D}^{1.4}/\rho$ Popovics (1975)

Dynamic Testing → Static Testing → Analysis

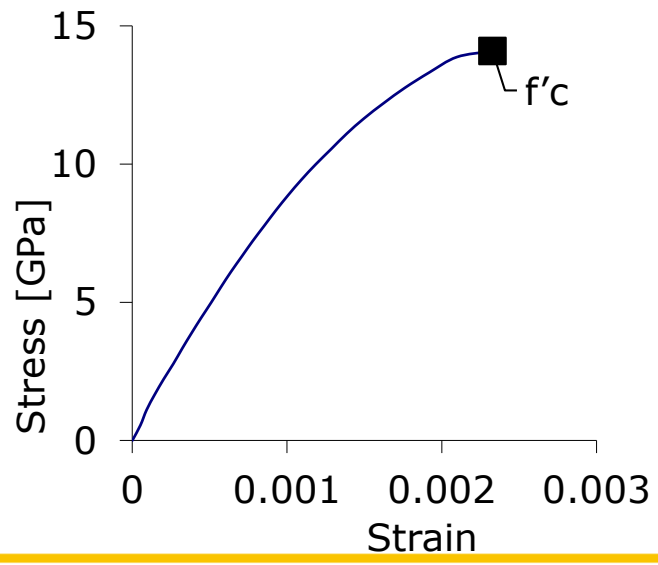
- 34 cylinder
- Dimensions
  - D ≈ 15 cm
  - h ≈ 30 cm

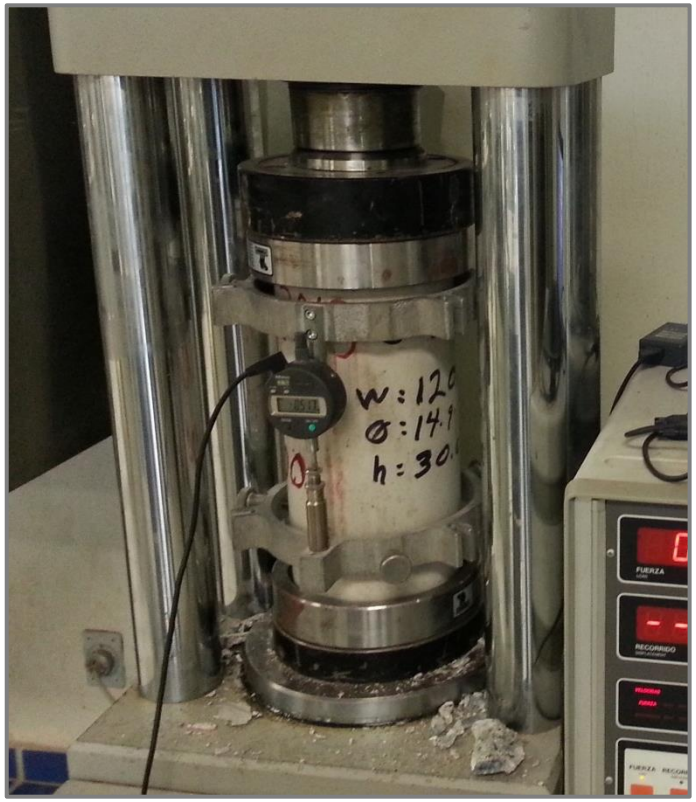
Dynamic Testing → Static Testing → Analysis

- Equipment
  - Hammer
  - Hydrophone
- 3 dynamic measures

Dynamic Testing | **Static Testing** | Analysis

- Equipment
  - Hydraulic Press
  - Strain Gauge





Dynamic Testing → Static Testing → **Analysis**

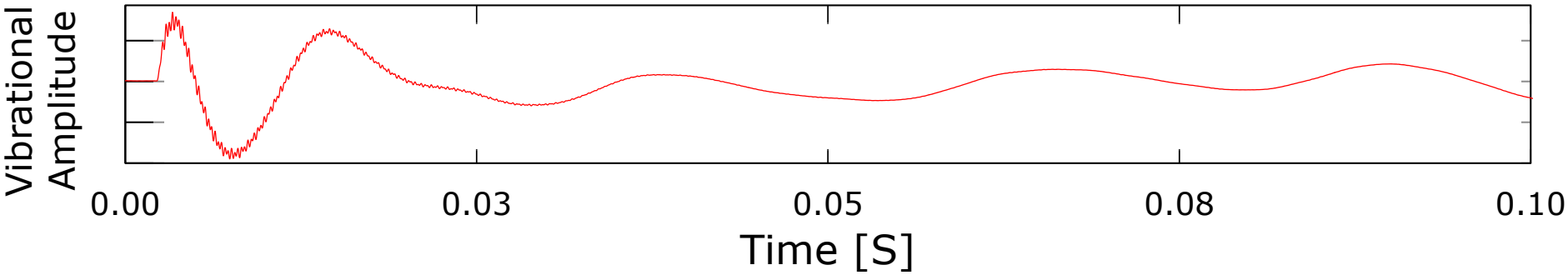
- Dynamic Analysis
  - Range Sweep:

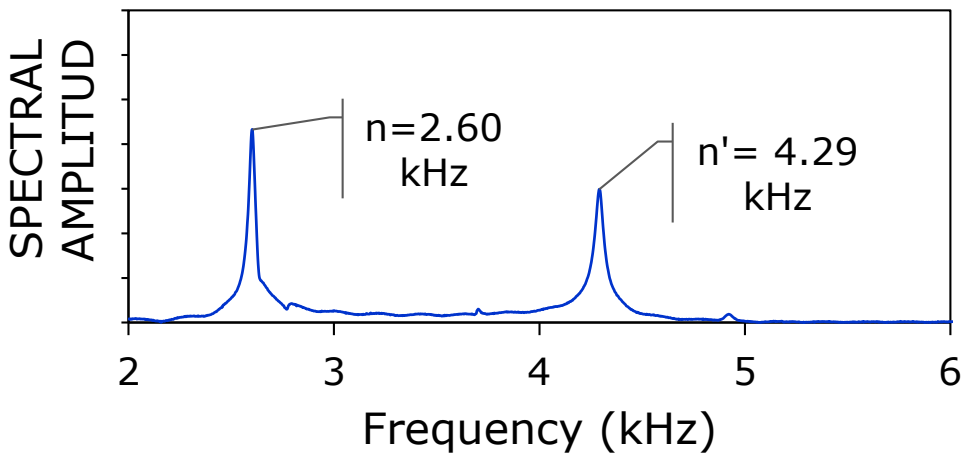
    2 kHz – 10kHz

  - Find peaks ratio:

$$f_t = 0.6125\, f_l$$

$$\begin{cases} C\, M\, {f_t}^2 = D\, M\, {f_l}^2 \\ f_t = 1.78041\, \dfrac{d}{L\sqrt{T}}\, f_l \end{cases}$$

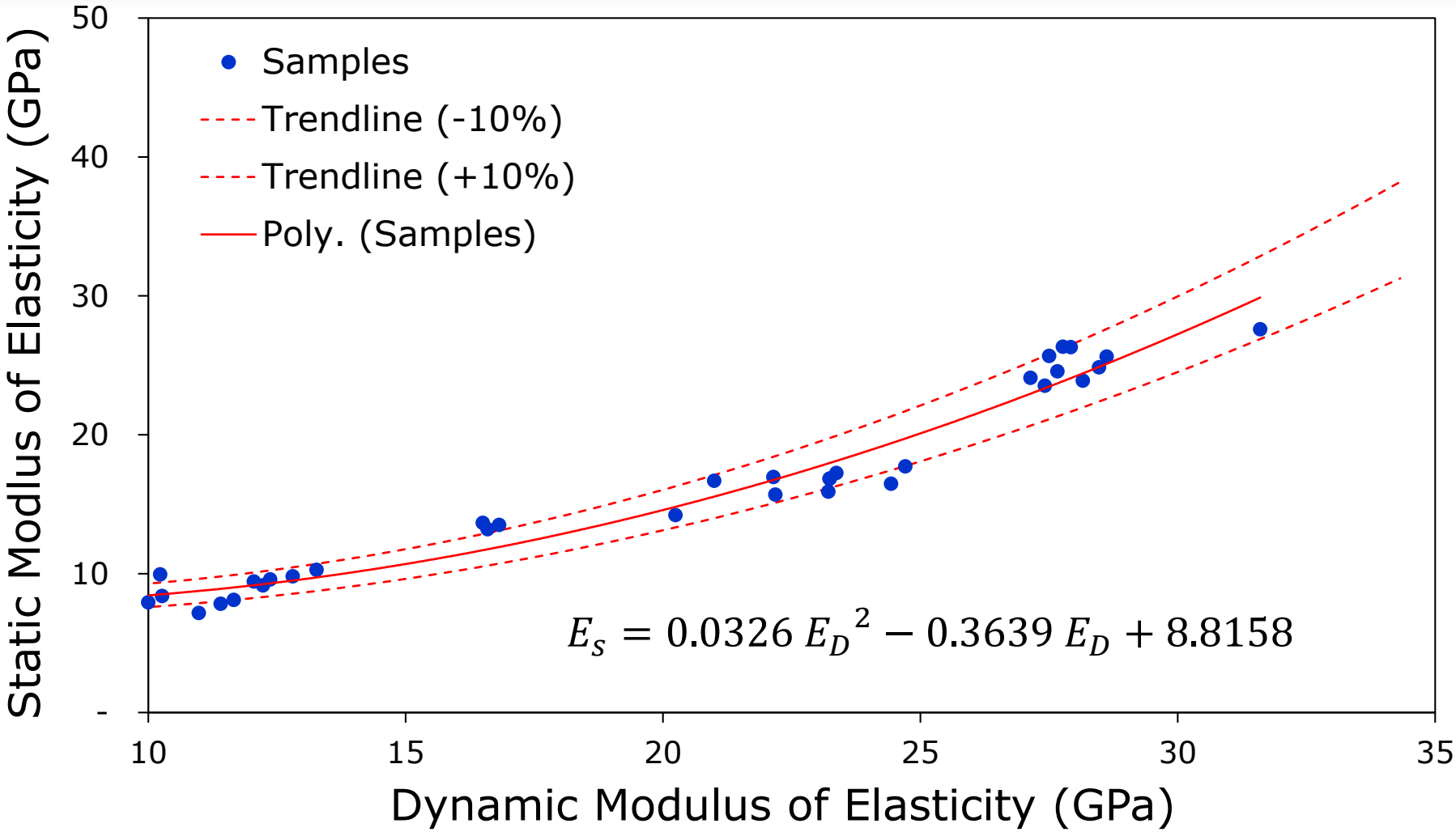




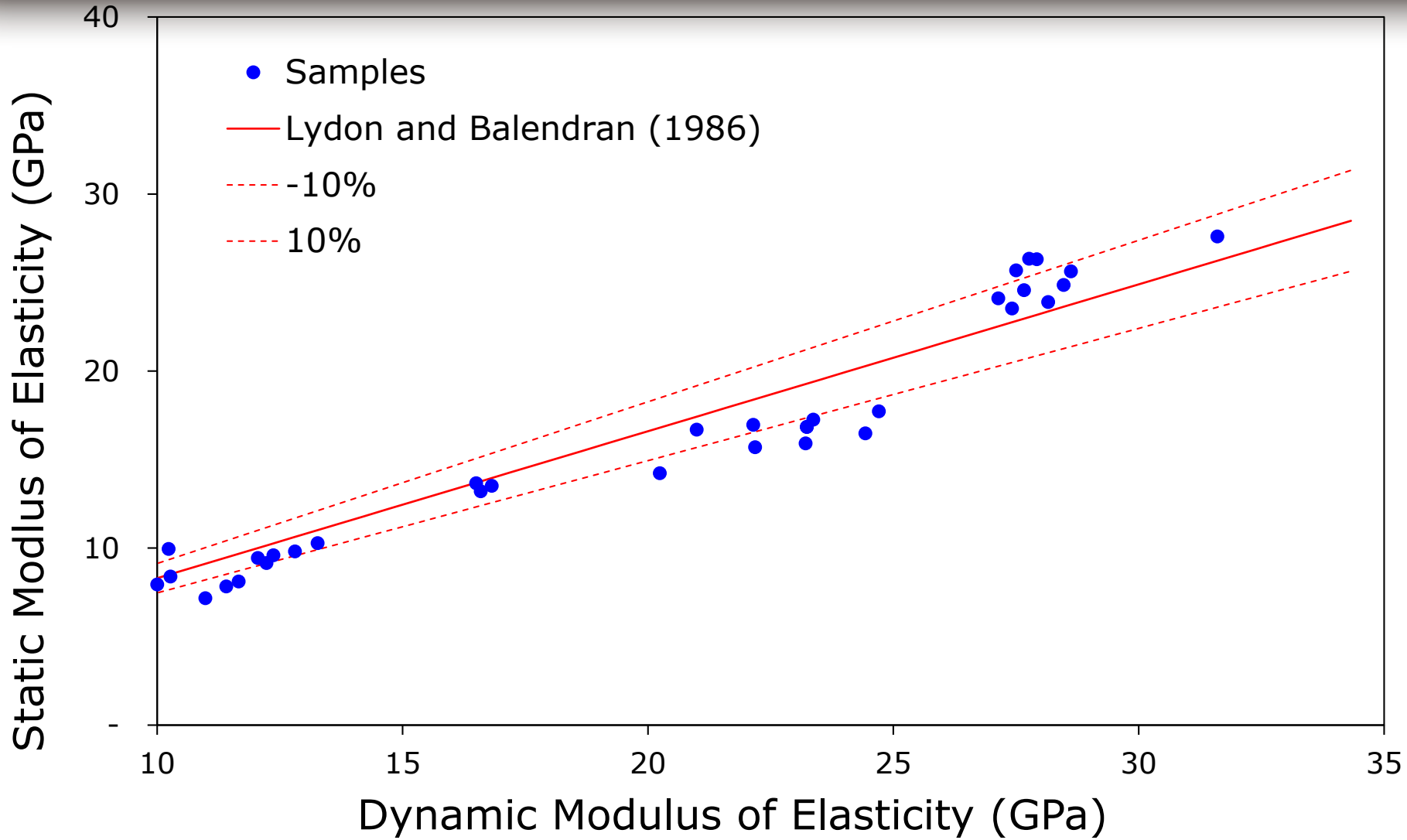
Fundamental Frequencies:

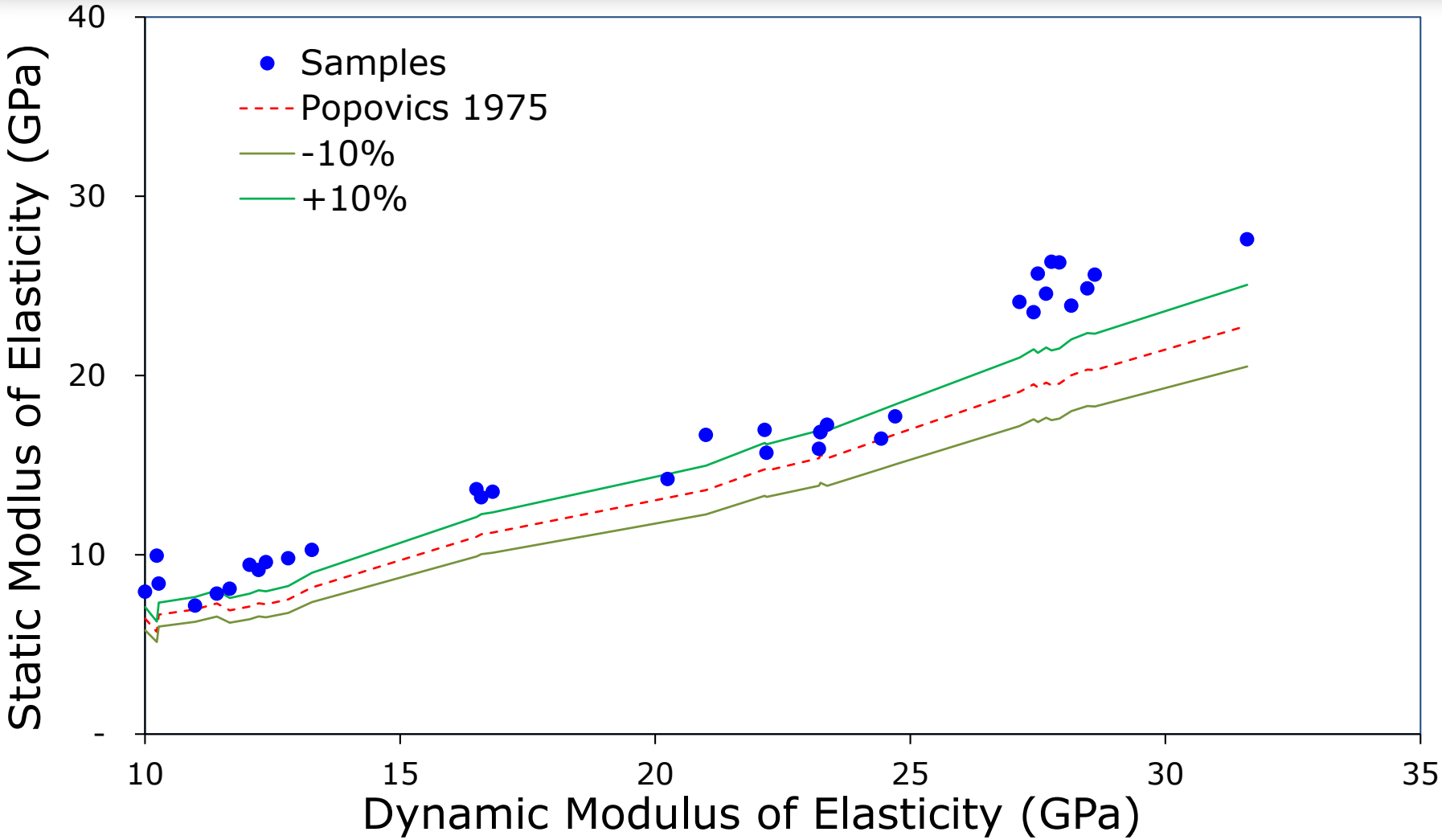
- Longitudinal ≈ 4.29 kHz
- Transversal ≈ 2.60 kHz

$$\frac{2.60}{4.29} = 0.606 \approx 0.6125$$


Samples
Trendline (-10%)
Trendline (+10%)
Poly. (Samples)
$E_s = 0.0326\,E_D{}^2 - 0.3639\,E_D + 8.8158$
Static Modlus of Elasticity (GPa)
Dynamic Modulus of Elasticity (GPa)
50
40
30
20
10
-
10
15
20
25
30
35

RESULTS


Samples
Lydon and Balendran (1986)
-10%
10%
Static Modlus of Elasticity (GPa)
Dynamic Modulus of Elasticity (GPa)
40
30
20
10
-
10
15
20
25
30
35


Samples
Popovics 1975
-10%
+10%
Static Modlus of Elasticity (GPa)
Dynamic Modulus of Elasticity (GPa)
40
30
20
10
-
10
15
20
25
30
35

- New correlation $E_S(E_D)$ proposed
  - Further study

- New methodology – More cost effective
  - No transducers
  - Using transverse/longitudinal ratio

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THANK YOU!

Questions?

REFERENCES

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